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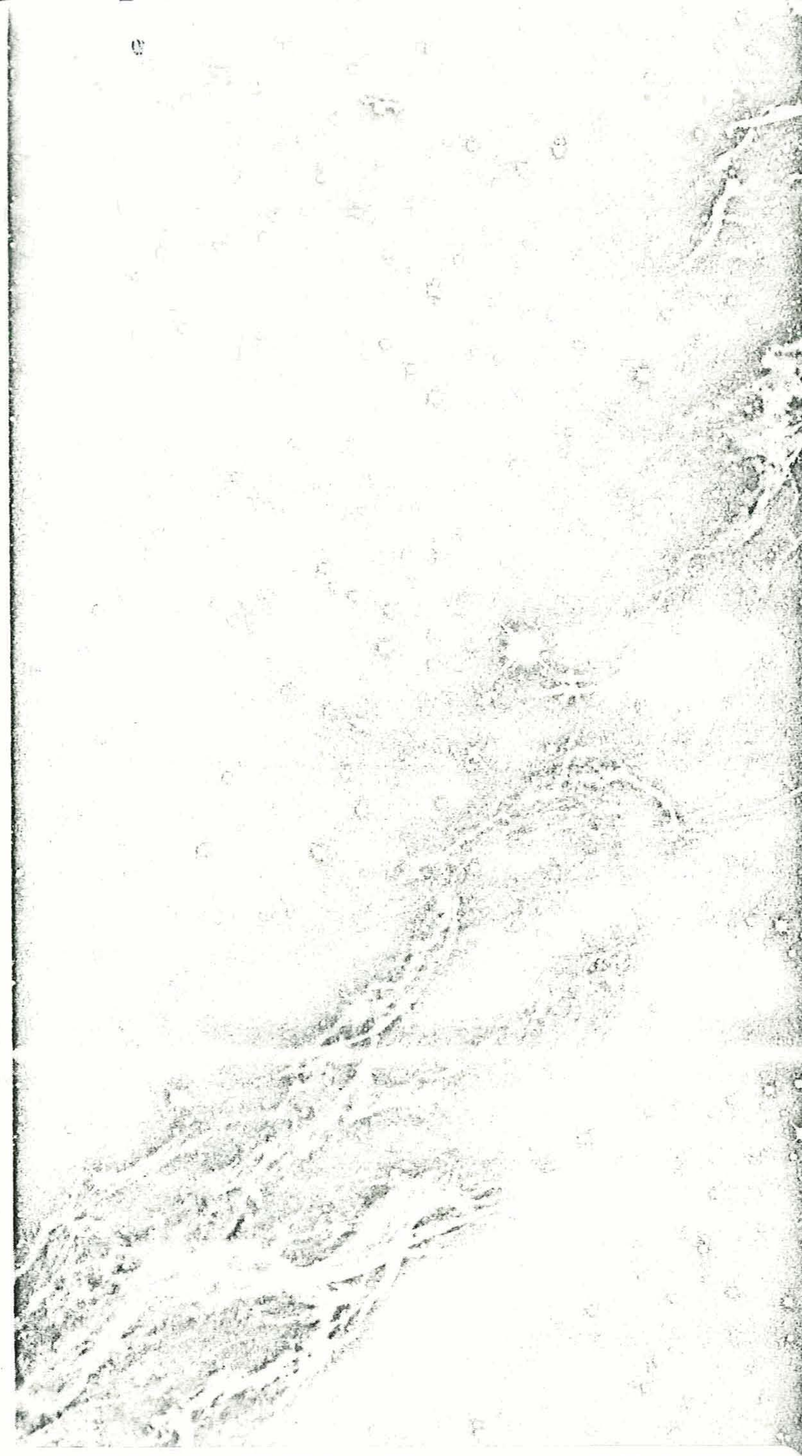

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THE PHYSICAL STRUCTURE AND IMPLIED NAVIGATIONAL HAZARD OF THE SATURN RING SYSTEM

IIT RESEARCH INSTITUTE
10 West 35 Street
Chicago, Illinois 60616

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THE PHYSICAL STRUCTURE AND IMPLIED NAVIGATIONAL HAZARD OF THE SATURN RING SYSTEM

by

M. J. Price
Astro Sciences

IIT Research Institute
Chicago, Illinois

for

Planetary Programs Division
Office of Space Science and Applications
NASA Headquarters
Washington, D. C.

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APPROVED BY:

D. L. Roberts, Manager
Astro Sciences

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SUMMARY

Planetary mission analysts are currently concerned by the potential hazard the Saturn ring system presents to spacecraft navigating near the planet. Uncertain knowledge of the physical properties of the ring system has caused a conservative attitude to prevail; for safety fly-by trajectories have been constrained to pass well outside the rings. Since any increase in our knowledge of the ring properties would broaden the range of practical options available to Grand-Tour missions, further study has been made of its structure.

Earlier dynamical and photometric studies of the physical structure of the ring system have been subjected to detailed criticism. On the basis of that critique, the best of the available photometric data have been re-interpreted in terms of the Mie theory of scattering. A new ring model has been developed; the surface density (i.e. the fraction of the ring plane covered by particles in a perpendicular projection) appears to be less than unity throughout, and the particles appear to be ice crystals of characteristic radius $\sim 0.1\mu$. No reliable information has been obtained concerning the probable size-distribution of the particles.

Caution must be exercised in estimating the implied navigational hazard since the ring model cannot be considered final. Even so, the probable effect on a TOPS-class spacecraft, intersecting the densest region of the ring system, while on a 4-planet (JSUN) Grand-Tour mission, has been studied. Individually, particle impacts should cause negligible damage since the TOPS design calls for the spacecraft to withstand impacts by particles of mass less than 10^{-2} gm striking at 20 kms sec^{-1}; the ring particles are of mass $\sim 10^{-15}$ gm, while the relative spacecraft-particle velocity is ~ 10 km sec^{-1}. Collectively, the ring particles will reduce the spacecraft velocity by ~ 0.4 m sec^{-1}

through interchange of momentum. On the basis of current knowledge, the ring system probably does not represent a significant hazard for Grand-Tour missions.

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1. INTRODUCTION

During the decade of the 70's the United States will launch unmanned scientific missions towards every planet in the solar system. Current plans to send spacecraft on multiple planet tours beyond Jupiter have prompted a resurgence of scientific interest in the outer planets. Long neglected, their study is expected to be a cornerstone in our understanding of both the origin and the evolution of the solar system. Of particular interest is the Saturn ring system, a unique feature of the solar system whose very existence demands explanation. Undoubtedly, high priority will be given to the study of the rings in any Saturn mission.

Study of the physical structure of the ring system is timely on several counts. First, planetary mission analysts are currently concerned by the potential hazard the rings present to spacecraft navigating near the planet. Uncertain knowledge of the physical properties of the system has caused a conservative attitude to prevail. For safety, fly-by trajectories have been constrained to pass well outside the rings. Any increase in our knowledge of the ring properties would broaden the range of practical options available for Grand-Tour missions. Second, before any spacecraft experiments are proposed for the study of the rings, it is essential that all information possible has been gleaned from ground-based observations. Only then can the scientific value of any Saturn mission be maximized.

New research on the physical structure of the ring system is presented in this paper. Section 2 provides a review of earlier dynamical and photometric studies. A critique is given of ground-based photometric techniques for inferring the ring properties. Present knowledge of the ring structure is shown to be both meagre and unreliable. In Section 3, Mie scattering theory is used to derive a new ring model from the best existing photometric data. Section 4 contains a brief discussion of the potential hazard the ring system presents to spacecraft. The conclusions of the report are contained in Section 5.

2. REVIEW OF EARLIER STUDIES

Although the existence of the Saturn ring system has been known for 300 years, our understanding of its basic nature stems from the theoretical paper by MAXWELL (1859) on the stability of the system. His results indicated that neither a solid-ring hypothesis nor a liquid-ring hypothesis was tenable, and suggested that the rings consist of a multitude of individual particles, each one pursuing a separate orbit around the planet. SEELIGER (1887, 1893) showed that the liquid-ring hypothesis was invalid because of the reflection properties of the rings. The particle theory was confirmed spectroscopically by KEELER (1895) who observed a Doppler effect, corresponding to differential rotation, in the solar spectrum reflected by the rings. Detailed spectroscopic study of the rings was made by CAMPBELL (1896) who showed that, throughout the system, the orbits of the particles are Keplerian and nearly circular.

Since the turn of the century, the most significant research on the dynamics of the ring system has been carried out by COOK and FRANKLIN (1964, 1966). In a thorough rediscussion of Maxwell's classic paper, they derived an upper limit to the volume density of the rings. Their studies showed that the ring system should definitely be stable if the density is less than $0.18\ \mathrm{gm.cm^{-3}}$, but unstable if the density is greater than $1.04\ \mathrm{gm.cm^{-3}}$.

Using dynamical arguments, JEFFREYS (1947) pointed out that the ring system should have evolved into a single layer of particles spaced just sufficiently far apart that collisions no longer occur. Recently, FRANKLIN and COLOMBO (1970) developed a dynamical model for the radial structure of Saturn's rings. Assuming the ring system to be a collisionless monolayer of particles, they showed that its major radial structure features could be explained as the result of gravitational perturbations by the two satellites Mimas and Titan.

TABLE I

GROUND-BASED PHOTOMETRIC STUDY OF THE SATURN RING SYSTEM

TYPE OF MEASUREMENT	INFORMATION CONTAINED IN DATA
EDGE-ON VISIBILITY	DEGREE OF FLATNESS OF RING SYSTEM; CRUDEST UPPER LIMIT ON VERTICAL RING THICKNESS AND DIAMETER OF RING PARTICLES
STELLAR AND SATELLITE OCCULTATIONS	OPTICAL THICKNESS OF RING AS FUNCTION OF RADIAL AND LONGITUDINAL POSITION IN SYSTEM; DISTRIBUTION OF PARTICLES IN RING PLANE
SURFACE BRIGHTNESS AS FUNCTION OF:	
1. RADIAL AND LONGITUDINAL POSITION IN RING	DISTRIBUTION OF PARTICLES IN RING SYSTEM
2. PHASE ANGLE (SUN-SATURN-EARTH): RANGE $\pm 6°$	PHASE SCATTERING FUNCTION OF RING PARTICLES; MUTUAL SHADOWING BY RING PARTICLES
3. SOLAR ILLUMINATION ANGLE (SATURNICENTRIC DECLINATION OF SUN) RANGE $\pm 27°$	MUTUAL SHADOWING BY RING PARTICLES
POLARIZATION	SCATTERING PHASE FUNCTION OF RING PARTICLES; ALIGNMENT OF PARTICLES IN RING PLANE
SPECTROSCOPIC	COMPOSITION OF RING PARTICLES; SIZE OF PARTICLES IN RELATION TO WAVELENGTH OF INCIDENT RADIATION

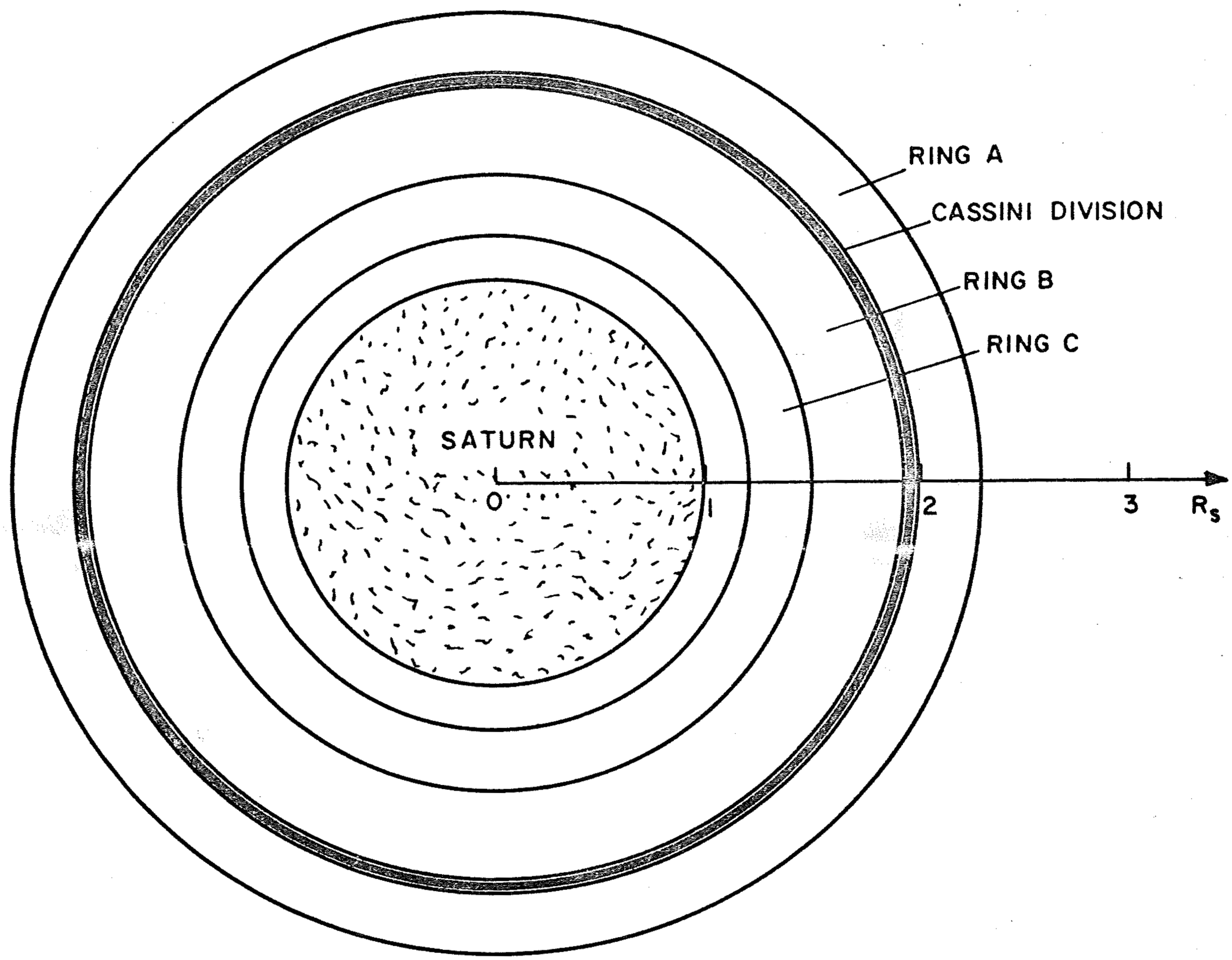


FIGURE 1. POLAR VIEW OF SATURN AND ITS RING SYSTEM
(R_s = EQUATORIAL RADIUS OF SATURN)

None of the dynamical studies has been able to provide reliable estimates of the physical properties of the individual ring particles. In the best attempt to date, FRANKLIN and COLOMBO (1970) derived a particle radius $\gtrsim$ 100 meters, but their ring model was highly idealized. As a result, until spacecraft carry out in situ sampling of the ring system, we must rely primarily on photometric studies to infer the physical properties of the individual particles. From Earth relatively few types of photometry can be applied to Saturn's rings. Table I summarizes the possibilities and the information to be gained from each. The different types of photometry are discussed more fully below. For subsequent reference, a schematic diagram of the ring system is presented in Figure 1. The three major components of the system, rings A, B and C, are shown together with Cassini's division. Dimensions of the system are taken from ALLEN (1963).

Observations of the ring system edge-on are possible every 15 years when the Earth passes through the ring plane. On a number of occasions during the past 100 years, measurements of the edge-on "visibility" of the rings have been used to infer their vertical thickness. Near the turn of the century, estimates of the thickness were derived from visual observations made by experienced observers using refracting telescopes of large aperture. Specifically, BARNARD (1891) obtained a thickness of less than about 75 km; RUSSELL (1908) obtained less than 21 km; and BELL (1919) obtained less than 15 km. For 50 years Bell's upper limit stood as the best estimate available. Recently, however, renewed attempts have been made to determine the ring thickness using photographic photometry. During 1966, when the rings were edge-on to the Earth, worldwide observations were carried out. Typical of the results obtained were those by KILADZE (1967) and by FOCAS and DOLLFUS (1969) who derived a ring thickness of 0.9 $\pm$ 0.6 Km. and 2.8 $\pm$ 1.5 km., respectively. But, in spite of the efforts made to measure the edge-on "visibility" accurately, the interpretation must necessarily be imprecise. Unless the rings are exactly planar, edge-on "visibility" will depend not only on the vertical thickness of the system but also in its degree of flatness. Edge-on observations can, therefore, provide only the crudest upper limits to both.

Observations of the occultations of stars, and of an eclipse of the satellite Iapetus by the ring system, have shown that all three of its components (rings A, B and C) are translucent. The available observations have been thoroughly reviewed by COOK and FRANKLIN (1958). Rough estimates of the optical thickness of the ring system have been obtained from stellar occultations, the most important occurring on 1920 March 14 when observers in the Union of South Africa (REID et al, 1920 a, b) and in India (BHASKARAN, 1920) followed the passage of BD + 11° 2269 behind rings A and B. Using a contrast argument based on the assumption that the diffraction disk of the star could be seen if its brightness were 1 percent of the surrounding ring, Cook and Franklin estimated the optical thickness of ring B to be 0.59. Reliable observations exist for only one other stellar occultation. On 1917 February 9, AINSLIE (1917 a, b) and KNIGHT (in AINSLIE and KNIGHT, 1917) observed the occultation of 212B Geminorum by ring A. During occultation, the star remained clearly visible to both observers, demonstrating the transparency of ring A. No useful estimate could, however, be made of its optical thickness. Cook and Franklin derived perhaps a better estimate for the optical thickness of ring B from observations by BARNARD (1890) of the 1889 November 1-2 eclipse of Iapetus by the shadow of the rings. They also determined the optical thickness of ring C as a function of radial distance from Saturn. In a detailed analysis of Barnard's observations, Cook and Franklin took into account not only the effects of the finite sizes of the sun and satellite, but also solar limb - darkening and the reflection properties of the satellite. Their best estimate of the optical thickness of ring B was 0.58, with a lower limit of 0.45 and an upper limit of infinity. DEIRMENDJIAN (1969) has criticised their analysis on the grounds that it depends on *ad hoc* assumptions concerning the reflectivity of the satellite, the unknown transmission properties of the ring itself, and the nature of the shadow it produces. Cook and Franklin also briefly discussed a study by BOBROV (1952) of the translucency of ring B which was based on the visibility of the ball of the planet through the ring. From analysis of photographic data obtained by CAMICHEL (1946),Bobrov derived an average optical thickness of ring B of 0.7. After discussing the original data with Camichel, Cook and Franklin concluded that Bobrov's interpretation was unsound. In principle, study of the visibility of the ball of the planet through the rings could provide useful information on the optical thickness of the system. So far, however, the available observational data are too imprecise for such a study to be useful.

In spite of the efforts made to reliably determine the optical thicknesses of the rings from stellar occultations, all analyses to date suffer from a fundamental weakness. All are based on "eye-ball" estimates of the brightness of the star against the bright background of the ring system. Such observations are notoriously difficult to make accurately. Being based entirely on the subjective judgement of the observer, they may well contain serious systematic errors. Observations of the eclipse of Iapetus must also be treated with reservation. Even though Barnard was unsurpassed as a visual observer, the satellite *is* difficult to see so near the planet because it is intrinsically very faint (apparent magnitude 10-12). Until such time as objective (photoelectric) photometry can confirm or replace the visual observations, we must assume that our current knowledge of the optical transmission properties of the ring system is very uncertain. In fact, at the present time, all we can say with certainty is that the rings are partially translucent. Unfortunately, occultations of bright stars are too rare for this situation to improve rapidly. Elementary considerations of the motion of Saturn against the stellar background indicate that the ring system occults a star brighter than apparent visual magnitude 8 only once in 20 years.

Many observations at visual wavelengths have been made of the variation of the surface brightness of the ring system as a function of radial distance from the planet. DOLLFUS (1961 b) has presented the most detailed and reliable results to date, based on both visual and photographic photometry of the rings. The brightness profile he obtained may be used to study the radial distribution of material throughout the ring system.

Measurements of the surface brightness as a function of phase angle have shown the existence of a pronounced phase effect, the rings brightening dramatically near opposition. FRANKLIN and COOK (1965) have determined two color (blue and yellow) phase curves for each of the bright rings, A and B, using photoelectric and photographic photometry. Their photometry is among the most accurate carried out on the ring system to date, and the phase curves they obtained may be considered definitive. Two distinct optical phenomena combine to produce the phase effect, mutual shadowing by the individual ring particles and their characteristic scattering phase functions. Franklin and Cook analyzed their observations using, both separately and in combination, a sophisticated model for mutual shadowing and a scattering model based on the Glory phenomenon. Their analysis produced two possible ring models. Model I consists of a ring of unknown thickness composed of particles of unknown radius with surface irregularities of characteristic size 7 μ; the fraction of the ring volume occupied by particles is $\sim 10^{-3}$. Model II consists of a ring of thickness between 3 cms and 10 cms, composed of particles of average radius $\sim 300\ \mu$; the fraction of the ring volume occupied by particles is $\sim 5 \times 10^{-3}$. Although Model I gave the better agreement with the observations, Franklin and Cook were unable to determine which, if either, model was correct. Fundamental to their analysis was evaluation of the relative contributions of mutual shadowing and scattering to the observed phase effect. But the relative importance of these phenomena cannot be determined unambiguously without a separate study of mutual shadowing alone, not possible at that time because of the lack of relevant observational data. Franklin and Cook's analysis also suffered because of incomplete treatment of the shadowing and scattering properties of small particles. Consequently, the results obtained were necessarily inconclusive, and the structure of the ring system remains very much an open question.

Potentially, measurements of the variation of the surface brightness of the ring system as a function of solar illumination angle can provide valuable information on the phenomena of mutual shadowing and multiple scattering by the individual particles. Published data by CAMICHEL (1958) and by FOCAS and DOLLFUS (1969), based on photographic photometry only, are meagre and imprecise. Nevertheless, LUMME (1970) has used these data to infer a maximum perpendicular optical thickness of the ring system (ring B) at visual wavelengths of 1.25. His analysis is open to question because, not only is the agreement between theory and observation poor especially at low solar illumination angles, but also the scattering theory used may be inappropriate. The theory of scattering by a finite, plane parallel, homogeneous layer is not strictly applicable if the ring particles are confined to a closely planar monolayer.

Polarization measurements of the ring system, carried out by LYOT (1929), have been reviewed by DOLLFUS (1961 a). At several apparitions of Saturn, Lyot measured the degree and direction of polarization of each of the bright rings, A and B, as a function of phase angle. His measurements from one opposition to the next were consistent for ring B, but not for ring A. Both ansae of ring B showed a slight variation of polarization with phase angle, the degree of polarization varying from zero (at zero phase) to never more than 0.5 percent. At all times the polarization was negative, the preferred plane of vibration being perpendicular to the plane of vision. New measurements secured by Dollfus since 1958 revealed more details. He found that the direction of polarization changed along the ring. For each value of the phase angle, the polarization could be assumed to be comprised of two contributions, one constant along the ring in amount and direction, the other rotating along the ring. The direction of the latter component was found to be either parallel or perpendicular to the tangent to the ring, depending on the phase angle. Dollfus attempted to explain this effect in terms of the orientation and surface structure of the ring particles. Recent experience by HALL (1970) suggests, however, that all these polarization measurements may be spurious. HALL and RILEY (1969) have shown that the light from the disk of Saturn is strongly polarized in the limb regions, less so nearer the center of the disk. They point out that light from the ball of the planet scattered in the optical system of the telescope contaminates polarization measurements of the ring. All polarization studies of the ring to date must, therefore, be treated with considerable caution.

Studies by PILCHER et al (1970) of the infrared solar spectrum reflected by the ring system have indicated that water-ice is a constituent of the individual particles. Recent high resolution spectrophotometry in the wavelength region 1-5 μ, carried out by KUIPER, CRUIKSHANK and FINK (1970 a, b), shows strong absorption bands corresponding to those produced by water-ice at the temperature of the ring material. Earlier low resolution studies of absorption features in the wavelength regions near 1 μ and 1.5 μ, made by OWEN (1965) and KUIPER (1952) respectively, had suggested that ice might be present in the rings. Data on the infrared reflectivity of the rings are not, however, sufficient to determine whether the ring particles are composed entirely of ice, or merely covered by a thin layer of frost.

Recently, LEBOFSKY, JOHNSON AND McCORD (1970) measured the spectral reflectivity of rings A and B in the wavelength range 0.3 - 1.05 μ, and interpreted their results in terms of compositional implications for the ring material. They found that the reflectivity decreases sharply towards blue and ultraviolet wavelengths, in the same manner for both rings. On the expectation that a pure water frost would have a flat reflection spectrum in the visual region, Lebofsky et al suggest that the ring particles must be composed of material other than pure ice, perhaps frost-covered silicates. Their interpretation did not, however, consider the influence of particle-size on spectral reflectivity.

Clearly, study of the optical properties of the ring system is still in its infancy. Very few accurate observational data exist, and almost none has been reliably interpreted. Of the various types of photometric observations made to date, the most useful for determining the physical structure of the ring system are measurements of the surface brightness as a function of solar illumination angle, phase angle, and radial distance from the planet. In the next section, the best of these data are used to develop a new ring model. Observations of the spectral reflectivity of the rings in the visual region are used to test the validity of the model.

3. RING MODEL

Three parameters describe the basic physical structure of the ring system:

1. The surface density, as a function of radial distance from the planet. Surface density is defined locally as the fraction of the ring surface physically covered by particles for a projection perpendicular to the ring plane.

2. The chemical composition of the particles.

3. The characteristic size, and size distribution of the particles.

Ring thickness is not included since it is necessarily an ill-defined parameter.

To perform a theoretical analysis of the optical scattering properties of the ring system, we must first adopt a description of the probable distribution of particles both in, and perpendicular to, the ring plane. From the dynamical studies discussed earlier, we will assume that the ring system consists of a uniform, planar monolayer of particles, and that the surface density is everywhere low (i.e. much less than unity). The perpendicular extinction optical thickness of the system will therefore be small throughout. So the rings should appear optically thin unless nearly edge-on to both the sun and Earth. Following the spectrophotometric detection of water-ice within the ring system, the analysis will be based on the assumption that the particles are clear ice-spheres of uniform but arbitrary radius.

The surface density, S, (i.e. the perpendicular geometrical optical thickness) can be written:

$$S = N . \Pi r^2/A \qquad (1)$$

where N is the total number of particles in the ring, A is the surface area of the system, and r is the radius of each particle. Further, we can write the apparent surface density, $S_{Apparent}$, (i.e. the perpendicular extinction optical thickness) as

$$S_{Apparent} = N . K_{EX} \Pi r^2/A = K_{EX} S \qquad (2)$$

where K_{EX} is the extinction efficiency of each particle. From the MIE (1908) theory of scattering, K_{EX} depends only on the refractive index and size of the particle in relation to the wavelength of the incident radiation.

Because of the probable geometrical distribution of ring particles, we can confidently expect both mutual shadowing and multiple scattering to be insignificant everywhere in the system when the solar illumination angle, θ, is large. Primary scattering alone should then provide an adequate description of the manner in which the ring system scatters the incident sunlight. Since Franklin and Cook observed in 1959 when the ring system was wide open ($\theta \sim 26°$), the phase effect they recorded must have resulted almost if not entirely from the scattering phase function of the individual particles. Supporting evidence that this is so comes from Franklin and Cook's own analysis of their observations. They found that although the surface brightnesses of rings A and B were very different, the ratio was independent of both phase angle and wavelength. Further evidence is provided by the measurements of LEBOFSKY, JOHNSON and McCORD (1970) of the spectral reflectivities of ring A and B. Both rings showed the same variation of reflectivity with wavelength. Both sets of observations may be explained in terms of the scattering properties of the individual particles if the ring system were everywhere optically thin, and if the particles have the same shape, composition and size-distribution in both rings. Surface brightness must then be directly proportional to the local concentration of particles; differences in the surface brightness of each ring will result from differences in the surface density. The FRANKLIN and COOK (1965) photometry of the phase effect may, therefore, be used directly to infer the basic optical scattering properties of the individual ring particles.

To show how the optical properties of the individual particles can be so inferred, we will begin by relating the monochromatic scattering efficiency of the system as a whole, to ground-based photometry of the phase effect. Denoting the monochromatic solar flux at Saturn as $F^{\odot}_{Saturn}$, the monochromatic luminosity of the sun as $L_{\odot}$, and the Saturn-sun distance as R_{Saturn}, we have

$$F^{\odot}_{Saturn} = L_{\odot}/4\,\Pi\,R^2_{Saturn} \tag{3}$$

Similarly, if $F^{\odot}_{Earth}$ is the monochromatic solar flux at the Earth, and R_{Earth} is the Earth-sun distance, we have

$$F^{\odot}_{Earth} = L_{\odot}/4\,\Pi\,R^2_{Earth} \tag{4}$$

Denoting the total monochromatic radiation flux striking the ring as $\mathfrak{F}^{\odot}_{Saturn}$, and the Saturnicentric declination of the sun as θ, we have

$$\mathfrak{F}^{\odot}_{Saturn} = F^{\odot}_{Saturn} \cdot A. \sin\theta \tag{5}$$

where, as before, A is the surface area of the ring system. If the total monochromatic radiation scattered by the ring is denoted by L_{Ring}, we have

$$L_{Ring} = \mathfrak{J}^{\odot}_{Saturn} \cdot Q \tag{6}$$

where Q is the monochromatic Bond albedo of the ring (i.e. the fraction of the incident radiation scattered into all solid angles by the ring material). Since it is unlikely that the ring will scatter the incident radiation isotropically, we will define the "gain", G, in the direction of the Earth as

$$G = \frac{\text{Intensity of radiation scattered towards the Earth}}{\text{Intensity if the radiation were isotropically scattered}} \tag{7}$$

Denoting the monochromatic Saturn ring flux at the Earth as F^{Ring}_{Earth}, and the Saturn-Earth distance as Δ, we can write

$$F^{Ring}_{Earth} = G \cdot \frac{1}{4\Pi\Delta^2} \cdot L_{Ring} \tag{8}$$

Introducing the apparent monochromatic magnitudes of the sun and the ring system as $m_{\odot}$ and m_{Ring}, respectively, we can write their difference as

$$\delta m = m_{Ring} - m_{\odot} \tag{9}$$

Using the definition of the magnitude scale, we can rewrite equation (9) as

$$\delta m = 2.5 \log_{10} \left[\frac{F^{\odot}_{Earth}}{F^{Ring}_{Earth}} \right] \tag{10}$$

Utilizing equations (3) through (10), we can show that

$$GQ = \frac{4\Pi \; \Delta^2 \; R^2_{Saturn}}{R^2_{Earth} \cdot A \cdot \sin\theta \; 10^{\,0.4\;\delta m}} \tag{11}$$

Equation (11) enables us to calculate the product of the gain and Bond albedo for the ring system, as a function of phase angle, directly from photometry of the phase effect. The "scattering product", GQ, applies not only to the ring system as a whole but, on average, to each element of its surface area.

In deriving equation (11), we have tacitly assumed that the surface density is unity. But, when the rings are widest open, we expect the line-of-sight optical thickness to be everywhere so low that only a fraction of each element of surface area projected on the plane of the sky is apparently covered by particles. Both mutual shadowing and multiple scattering are expected to be insignificant, so that each particle may be considered to scatter the incident radiation independently of its neighbors. Consequently, the "scattering products" of each element of surface area and of each ring particle are then directly related through the surface density. Denoting the "gain" of each particle by g, and the single scattering albedo by q, we have

$$gq = \frac{GQ}{S} \tag{12}$$

where, as before, S is the surface density. Using equation (2), we can rewrite equation (12) as

$$\frac{gq}{K_{EX}} = \frac{GQ}{S_{Apparent}} \tag{13}$$

Using an obvious notation, we can rewrite equation (13) for the special case of zero phase angle as

$$\frac{g_o q}{K_{EX}} = \frac{G_o Q}{S_{Apparent}} \tag{14}$$

But, we know that

$$q = \frac{\sigma_{sca}}{\sigma_{sca} + \sigma_{abs}} = \frac{\sigma_{sca}}{\sigma_{Ext}} \tag{15}$$

where σ_{sca}, σ_{abs} and σ_{Ext} are, respectively, the scattering, absorption, and extinction cross-sections of the particle. Using the scattering and extinction efficiencies, K_{sca} and K_{EX} respectively, defined by DEIRMENDJIAN (1969), we have

$$q = \frac{K_{sca}\ \sigma}{K_{EX}\ \sigma} = \frac{K_{sca}}{K_{EX}} \tag{16}$$

where σ is the geometrical cross-section of the particle. In the special case of zero phase angle, the "gain" of the particle can be written

$$g_o = \frac{K_b}{K_{sca}} \tag{17}$$

where K_b is its backscattering efficiency defined by DEIRMENDJIAN (1969) in terms of the "radar cross-section, $K_b\sigma$. Equation (14) may now be rewritten as

$$\frac{K_b}{K_{sca}} \cdot \frac{K_{sca}}{K_{EX}} \cdot \frac{1}{K_{EX}} = \frac{G_o\ Q}{S_{Apparent}} \tag{18}$$

or

$$\frac{K_b}{K_{EX}^2} = \frac{G_o\ Q}{S_{Apparent}} \tag{19}$$

Equation (19) is fundamental in our determination of the characteristic size of the ring particles. It relates the basic scattering parameters of each ring particle directly to ground-based photometry of the ring system. Even for the densest region of the ring system, we have reason to believe that

$$S_{Apparent} \ll 1 \tag{20}$$

so that equation (19) can be rewritten as the inequality

$$\frac{K_b}{K_{EX}^2} \gg G_o\ Q \tag{21}$$

Photometric data on the variation of the apparent visual magnitude of the ring system with phase angle, obtained by FRANKLIN and COOK (1965), may be used to derive the corresponding variation of the scattering product, GQ, for ring B. Attention is concentrated on ring B since it is the dominant feature of the ring system. Due allowance must be made for the fractional contribution ring B makes to the brightness of the entire ring system. Weighting of the data must include use of the appropriate surface area of ring B. Corrections for changes in the Saturn-Earth distance are not necessary; Franklin and Cook adjusted all their photometric data to refer to the distance at opposition, which occurred during the night of 1959 June 25/26.

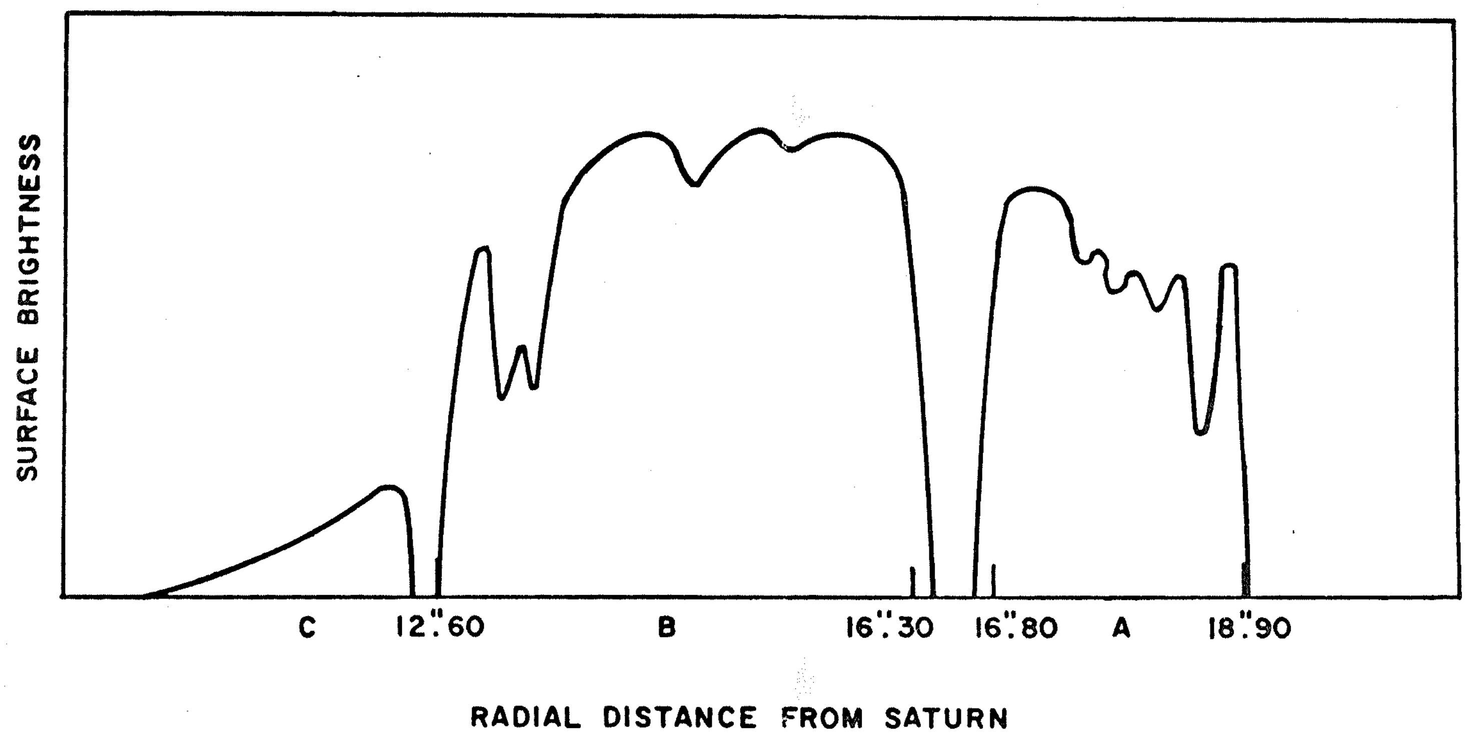


FIGURE 2. SURFACE BRIGHTNESS OF THE RING SYSTEM V RADIAL DISTANCE FROM THE CENTER OF SATURN (AFTER DOLLFUS (1961b)). ANGULAR DISTANCES REFER TO THE CASE WHERE SATURN IS LOCATED AT 10 A.U.

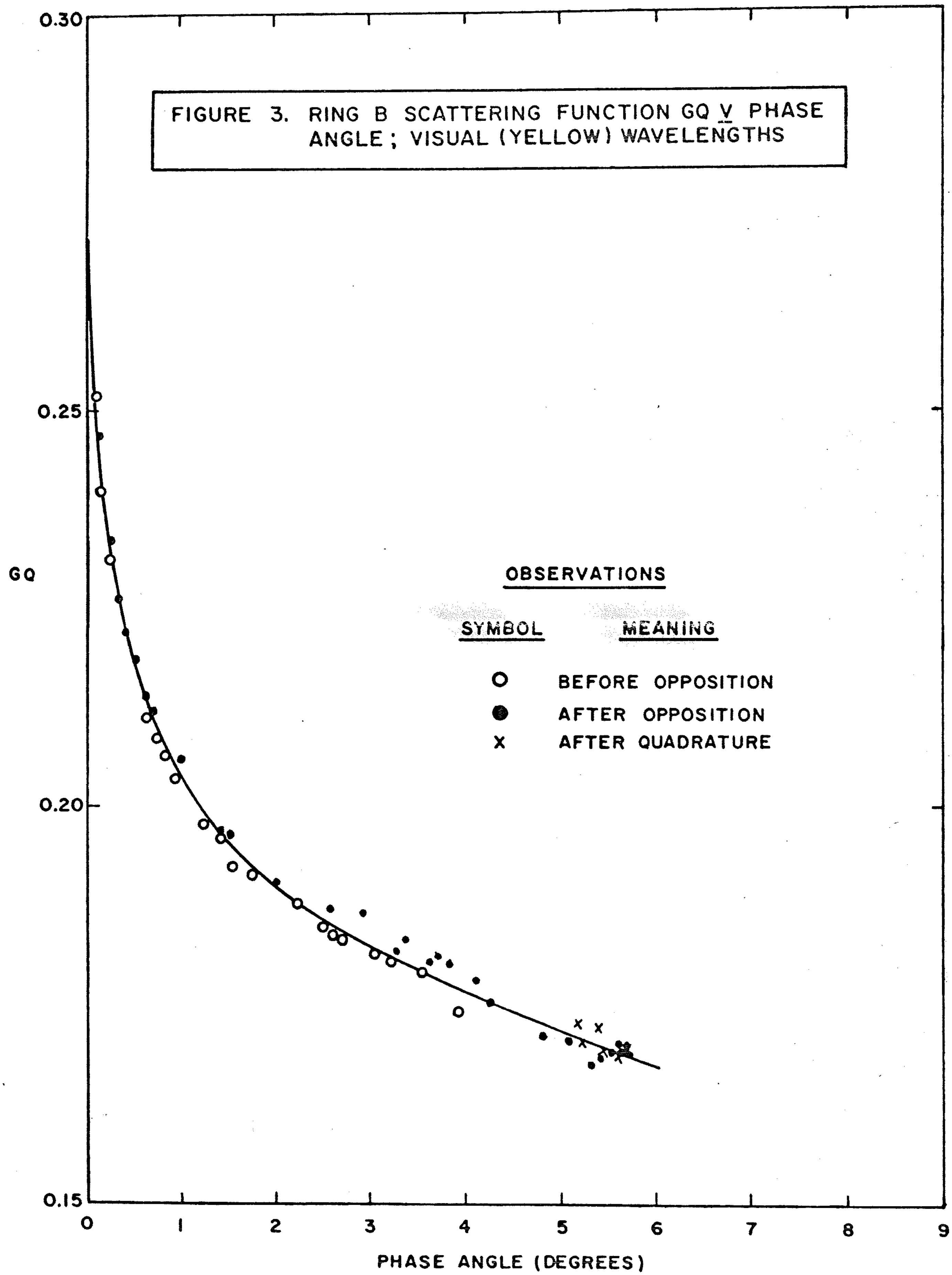


FIGURE 3. RING B SCATTERING FUNCTION GQ V PHASE ANGLE ; VISUAL (YELLOW) WAVELENGTHS
0.30
0.25
0.20
0.15
GQ
OBSERVATIONS
SYMBOL
MEANING
BEFORE OPPOSITION
AFTER OPPOSITION
X
AFTER QUADRATURE
0
1
2
3
4
5
6
7
8
9
PHASE ANGLE (DEGREES)

The fractional contribution ring B makes to the brightness of the entire ring system may be obtained from the radial profile of surface brightness at visual wavelengths given by DOLLFUS (1961b), and reproduced here in Figure 2. The radial profile Dollfus obtained is of direct relevance to the Franklin and Cook observations, since both sets of data were obtained for similar conditions of shadowing within the ring. The contributions of the component rings to the total brightness of the system are in the ratios Ring A: Ring B: Ring C:: 0.335: 0.628: 0.038. The ratio of the brightness of ring A to that of ring B is 0.532; in comparison, FRANKLIN and COOK (1965) obtained 0.398 $\pm$ 0.003, but their value is in fact the ratio of the brightness of ring A to the sum of the brightnesses of rings B and C. The surface area of ring B was obtained using the radii of the inner and outer edges given by ALLEN (1963). The apparent visual magnitude of the sun was also taken from ALLEN (1963). Figure 3 shows the resultant phase angle variation of GQ for ring B at visual (yellow) wavelengths. A smooth curve may be drawn through the data and extrapolated without difficulty to zero phase angle. We obtain,

$$G_o\, Q = 0.275 \tag{22}$$

Using equation (21), we have

$$\frac{K_b}{K_{EX}^2} \gg 0.275 \tag{23}$$

From the MIE (1908) theory of scattering, the "scattering function", $\frac{K_b}{K_{EX}^2}$, for each ring particle depends only on its refractive index and on its radius in relation to the wavelength of the incident radiation. To relate the radius, r, of the

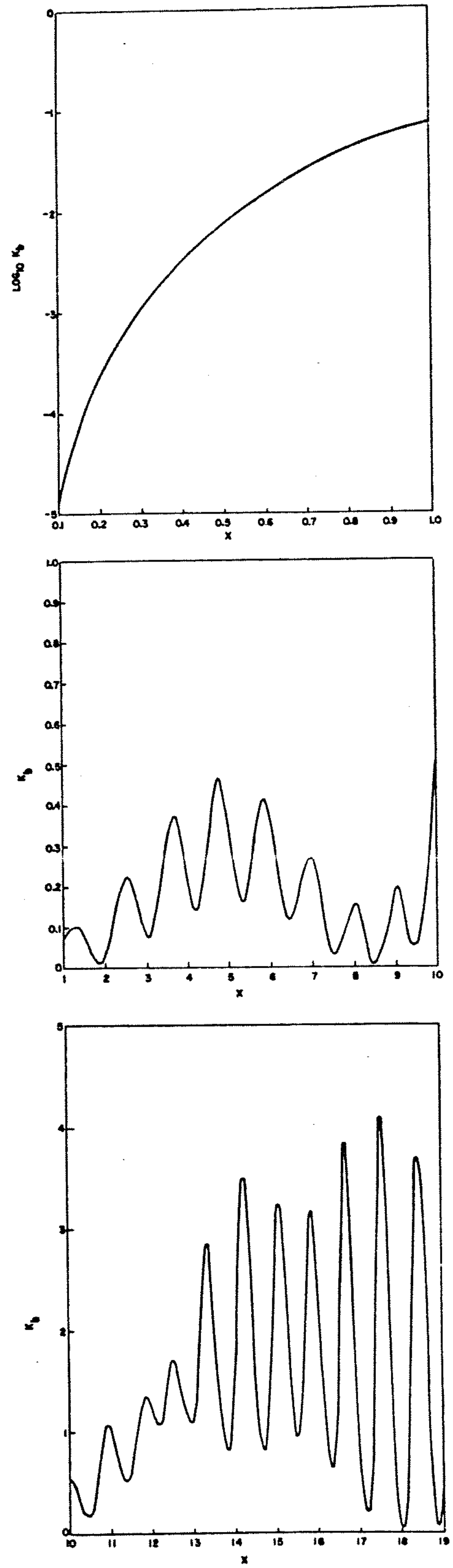
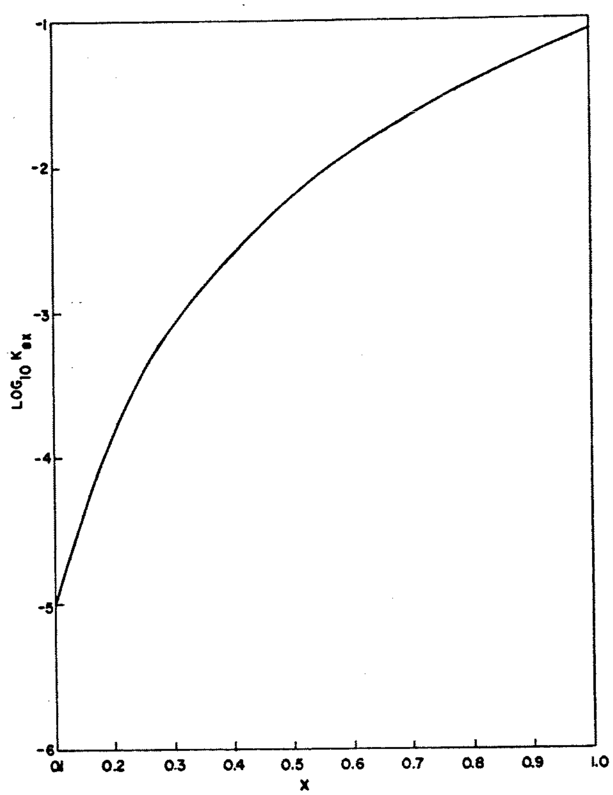


FIGURE 4. MIE SCATTERING FOR DIELECTRIC SPHERES OF REFRACTIVE INDEX 1.311 BACKSCATTERING EFFICIENCY, K_b V SIZE PARAMETER, X.



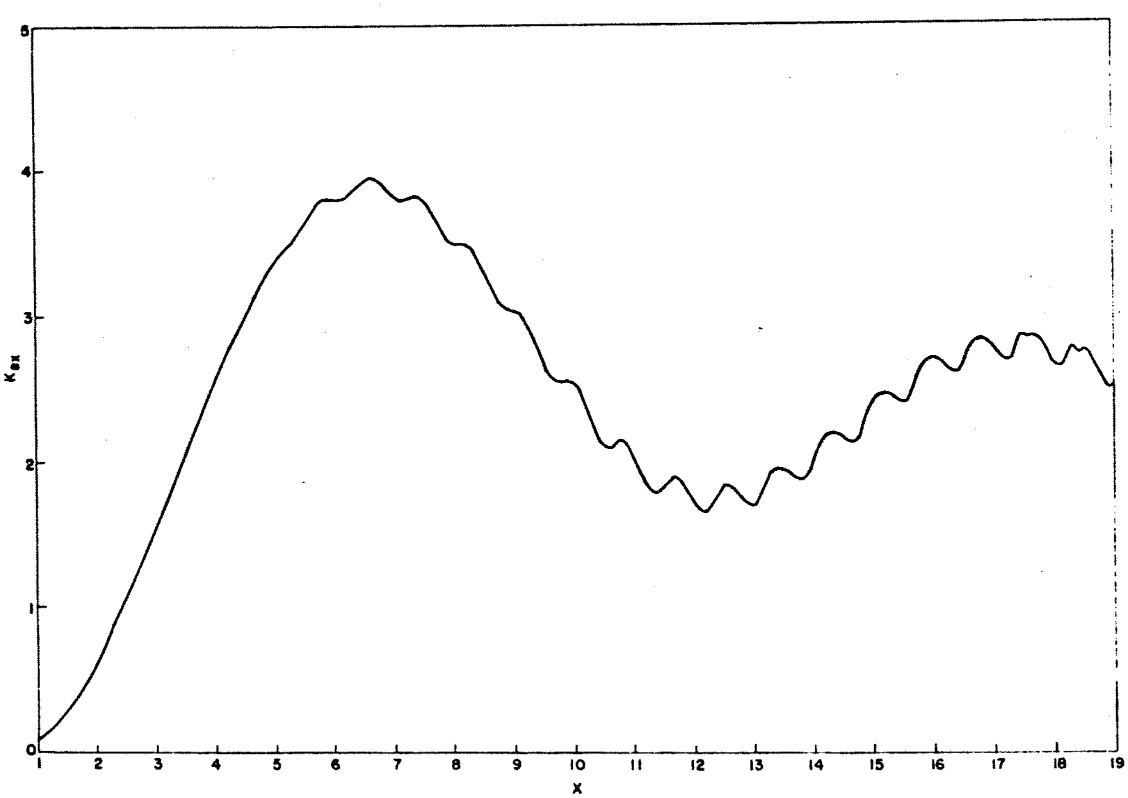


FIGURE 5. MIE SCATTERING FOR DIELECTRIC SPHERES OF REFRACTIVE INDEX 1.31: EXTINCTION EFFICIENCY, K_{EX}, V SIZE PARAMETER, X.

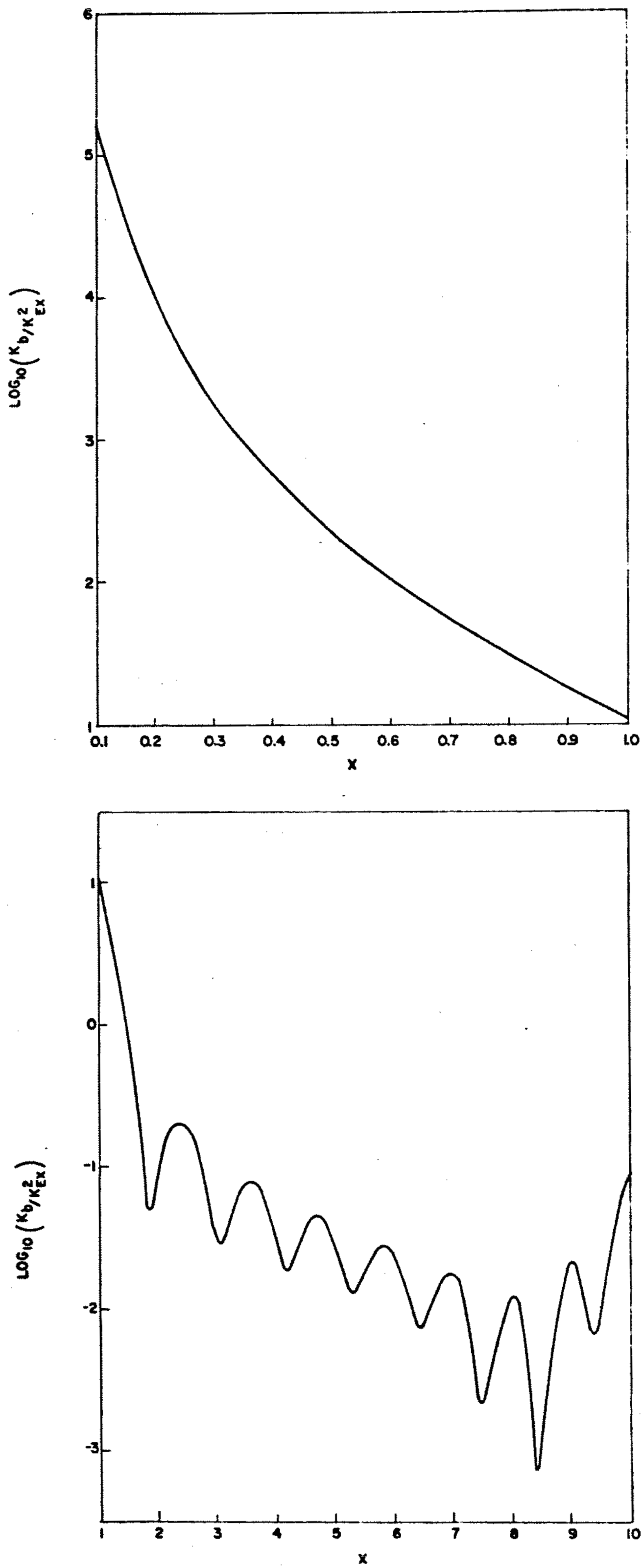


FIGURE 6. MIE SCATTERING FOR DIELECTRIC SPERES OF REFRACTIVE INDEX, 1.31: SCATTERING FUNCTION, K_b/K^2_{EX}, V SIZE PARAMETER, X.

particle to the wavelength, λ, of the incident radiation, a size parameter, x, is customarily defined, such that

$$x = 2\Pi r/\lambda \tag{24}$$

Since we have assumed that the particles are clear ice-spheres, the refractive index is already set at 1.31, the value for ice at a wavelength of 0.55 μ. The only free parameter is then the particle radius.

The Mie scattering parameters, K_b and K_{EX}, were calculated for dielectric spheres of refractive index 1.31 over the size range $0.01 < x < 100$ at Δx intervals of 0.01. The main results are plotted in Figures 4 and 5, for K_b and K_{EX} respectively. The quotient $\frac{K_b}{K_{EX}^2}$ is plotted in Figure 6 for $x < 10$. Large value of $\frac{K_b}{K_{EX}^2}$ correspond to smaller values of x and vice versa. For $x > 10$, $\frac{K_b}{K_{EX}^2}$ oscillates about an equilibrium value of 0.5. For $\frac{K_b}{K_{EX}^2} >> 0.275$, we require $x \lesssim 1.7$. For a wavelength of 0.55 μ, equation (24) gives an upper limit to the characteristic radius of the ring particles of 0.148 μ. Such a small radius implies that the apparent surface density is less than the geometrical surface density since for $x \leqslant 1.7$, $K_{EX} \leqslant 0.392$ (cf: equation (2)).

Measurements of the spectral reflectivity of ring B throughout the visual region, made by LEBOFSKY, JOHNSON and McCORD (1970), may be used to test the validity of the ring model. Theoretical predictions of spectral reflectivity can be restricted to the case of zero phase angle, since Lebofsky et al found no appreciable phase variation in their observations. Because the rings were observed during the 1969 apparition of Saturn, when the solar illumination angle was large ($\theta \sim 17°$),

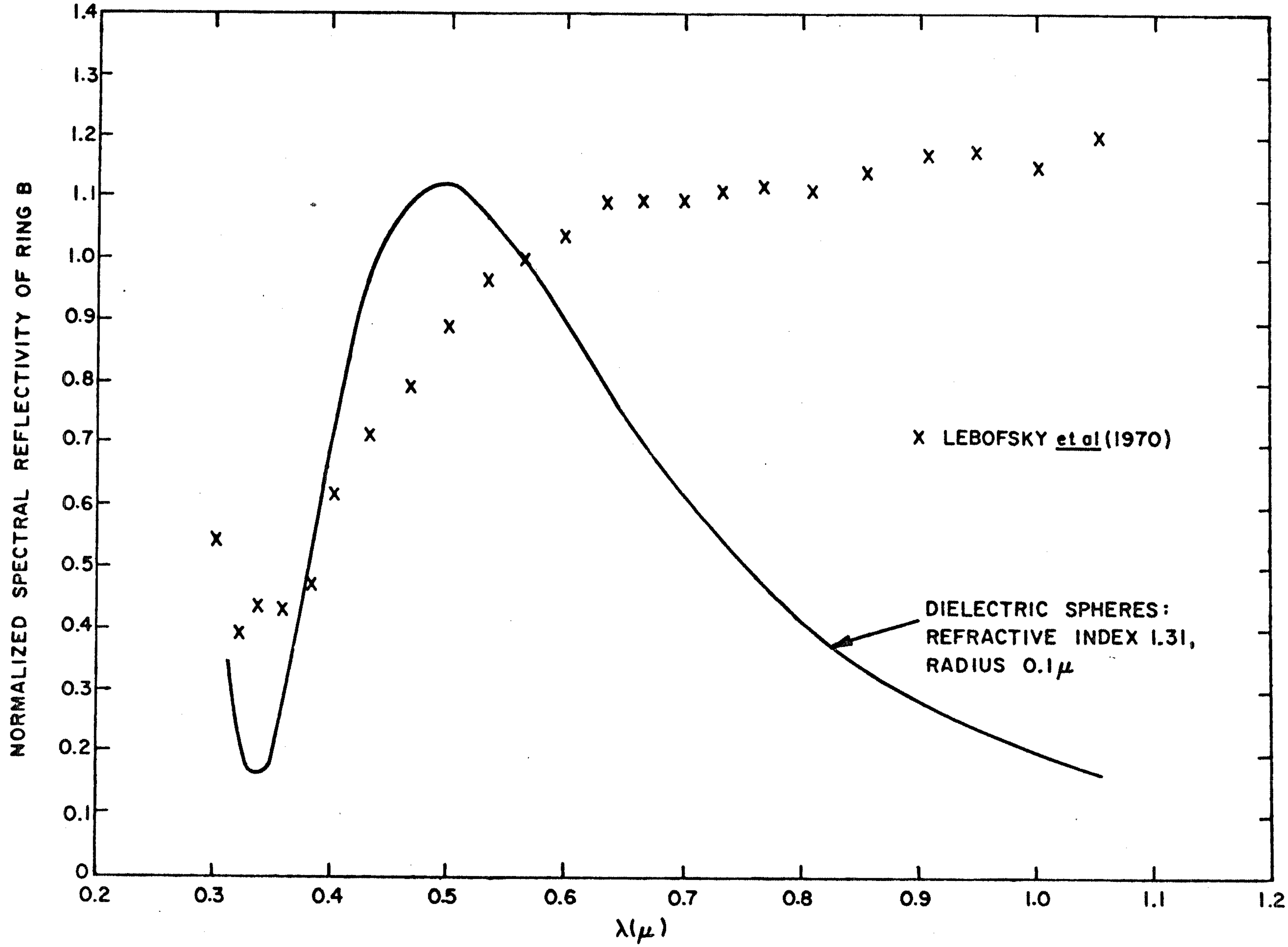


FIGURE 7. NORMALIZED SPECTRAL REFLECTIVITY OF RING B V WAVELENGTH: COMPARISON OF THEORY WITH OBSERVATION.

theoretical prediction of their spectral reflectivity is relatively straightforward; we may assume that the line-of-sight optical thickness at visual wavelengths was then low everywhere in the system. To a good approximation, the monochromatic surface brightness of ring B at zero phase angle then depended only on the total backscattering cross-section of the particles (cf: DEIRMENDJIAN (1969)). The spectral reflectivity should, therefore, follow the functional form of the variation with wavelength of their back scattering efficiency K_b. Theoretical predictions of the spectral reflectivity were made using the Mie theory for dielectric spheres of uniform radius and with refractive index 1.31. Radii in the range 0.05 μ - 0.2 μ were used. Best agreement between observation and theory was obtained for a radius of 0.1 μ. For very much larger particles, the spectral reflectivity curve was flat; for smaller particles, the decrease in reflectivity towards the ultraviolet was far more pronounced than was observed. In Figure 7, theoretical predictions for ring particles of radius 0.1 μ are compared with observation. Like the observations, the theoretical predictions have been normalized at λ 0.564 μ. Theory follows the general run of the observations for wavelengths less than $\sim$ 0.5 μ. Note that the suspected hook in the observations near 0.3 μ is predicted. Significant discrepancies between theory and observation do, however, occur in the red and infrared regions of the spectrum. The origin of these discrepancies will be pointed out below. Nevertheless, Figure 7 suggests that the observed decrease in the reflectivity of ring B towards blue and ultraviolet wavelengths is a consequence, not of absorption within the particles, but of their small size in relation to the wavelength of the incident radiation. There is no necessity to suppose that the ring particles are composed of material other than pure water ice. Figure 7 further suggests that the characteristic radius of the ring particles is $\sim$ 0.1 μ.

Undoubtedly the ring system contains particles with radii larger and smaller than the estimated characteristic value. That the rings might in fact be composed of a monolayer of predominantly very large particles, with small scale surface roughness, is suggested by the experimental discovery by OETKING (1966), and confirmed by HAPKE (1966), that many substances in widely varying conditions exhibit a pronounced brightness maximum in the backscattering direction. Large particles are, however, difficult to reconcile with our analysis. For particles with radii very much larger than the wavelength of light, Mie theory requires $\frac{K_b}{K_{EX}^2} \sim 0.5$, inconsistent with equation (23). However, if the ring system should not be optically thin, large particles cannot be entirely ruled out.

Three fragments of information suggest that particles with radii larger than the characteristic value are indeed present in the rings. First, the pronounced phase effect at visual wavelengths cannot be explained by the Mie theory if the ice-particles are of uniform radius 0.1 μ; particles with radii in the range 1 μ - 10 μ are needed. One should bear in mind, however, that small ice crystals are unlikely to be perfect spheres. Consequently, Mie theory may not be strictly applicable. Certainly, the characteristic radius of the ring particles is not so small that their shape will not affect their scattering properties at visual wavelengths. It is, therefore, conceivable that the observed phase effect may result from the shape of the particles rather than from their size-distribution. Second, the spectrophotometric detection of strong infrared band absorption characteristic of water-ice seems to require the presence of particles with radii greater than $\sim$ 0.1 μ; CRUIKSHANK (1971) has pointed out that such strong bands cannot be produced in the laboratory unless the scattering ice-particles have radii greater than $\sim$ 1 μ. Third, Figure 7 indicates that, for Mie theory to accurately predict the spectral reflectivity of the rings in the red and infrared region, the ring model must include some particles with radii $\sim 1\ \mu$. Micron-size particles may, therefore, exist within the ring system. If indeed they do, particles with radii smaller than the characteristic value must also be present to counterbalance their optical effects.

In principle, interpretation of the phase effect and spectral reflectivity curve should enable us to derive the probable size distribution of the particles. Unfortunately, the currently accessible range in phase angle is insufficient for useful information to be obtained, primarily because the functional form of the distribution must necessarily be a matter for sheer speculation. Further difficulties arise in the interpretation if the particles are non-spherical. Additional spectral reflectivity measurements are also required in the ultraviolet and infrared. No attempt has, therefore, been made to derive the size distribution of the particles.

Serious questions arise concerning the longevity of the ring system if the particles are ice crystals of characteristic radius $0.1\ \mu$. Although the rings would then be stable against the disruptive influence of radiation pressure, effects of sputtering and thermal evaporation must be considered. HARRISON and SCHOEN (1967) have pointed out that sputtering by ultraviolet radiation and proton bombardment would erode several, and possibly many centimeters of material from the surfaces of the particles in several billion years. By way of comparison, OWEN (1965) has shown that thermal evaporation is relatively insignificant. Meteoroidal bombardment must also be considered. Studies by COOK and FRANKLIN (1970) and by BANDERMANN and WOLSTENCROFT (1969) suggest that meteoroidal bombardment may have drastically altered the physical structure of the ring system over the age of the solar system. Probably, a steady and ultimately overwhelming accumulation of meteoritic material may have occurred; some erosion of the original ring material must certainly have taken place. But whether accumulation or erosion dominated remains uncertain. In-elastic collisions between incident meteoritic matter and the original ring material could, through erosion, have caused a general reduction in the characteristic radius of the particles. Particle-particle collisions within the ring system may have resulted in surface erosion also. In this connection, recent studies of the dynamics of the ring particles, by FRANKLIN and COLOMBO (1970), indicate that particle-particle collisions will occur provided that the particle radii are less than $\sim$ 100 meters. On the basis of these considerations, it appears that the physical structure of the rings may have evolved considerably over the lifetime of the system. Indeed, the structure may still be evolving. That the present state of the ring system appears to be an ensemble of tiny ice crystals may, therefore, not be so surprising; it may simply be the result of 4.5 billion years of evolution. Whether in fact the characteristic radius of the ring particles <u>has</u> evolved is a question that can be answered only after a thorough knowledge of the size-and shape-distributions of the particles has been obtained. At the present time, it appears that such knowledge can be obtained only by <u>in situ</u> sampling of the rings.

4. NAVIGATIONAL HAZARD

The Saturn ring system presents a potential navigational hazard; multiple collisions with the ring particles might severly damage a spacecraft passing through the ring plane. To obtain a conservative estimate of the seriousness of the hazard, it will be assumed that the spacecraft intersects the densest region of the ring system (ring B).

Of fundamental significance in evaluating the hazard are:

1. The characteristic radius, r, and individual mass, m, of the ring particles.
2. The total number, n, of particles projected on unit area of the ring plane.
3. The integrated mass, M, of the particles per unit area of the ring plane.

On the assumption that the ring particles are spherical and of uniform size, the characteristic individual mass is given by

$$m = \frac{4}{3} \Pi r^3 \rho \qquad (25)$$

where ρ is the density of the material in each particle.

Since we have reason to believe the surface density, S, is less than unit throughout the ring system, the total number of particles projected on unit area of the ring plane is, quite generally, given by

$$n = \frac{S}{\Pi r^2} \qquad (26)$$

The integrated mass of the particles per unit area of the ring plane is given by

$$M = n_{\circ} \; m_{\circ} \tag{27}$$

Substituting equations (25) and (26) into equation (27), we obtain

$$M = \frac{4}{3} S. \; r. \rho \tag{28}$$

Substituting the basic parameters of our ring model, namely,

$$\left. \begin{array}{lll} r & \sim & 0.1 \; \mu \\ \rho & = & 1 \text{ gm cm}^{-3} \\ S & < & 1 \end{array} \right\} \tag{29}$$

into equations (25), (26) and (28), we obtain

$$\left. \begin{array}{lll} m & \sim & 4.2 \times 10^{-15} \text{ gm} \\ n & < & 3.2 \times 10^{13} \text{ m}^{-2} \\ M & < & 1.3 \times 10^{-1} \text{ gm m}^{-2} \end{array} \right\} \tag{30}$$

On the basis of the model discussed in Section 3, the amount of material projected on each square meter of the ring plane is negligible. Although the number of particles per square meter is large ($\sim 10^{13}$), their average radius is small ($\sim 0.1\mu$) and their individual mass is tiny ($\sim 10^{-15}$ gm); the integrated mass per square meter of the ring surface is only ~ 0.1 gm.

To illustrate the apparent insignificance of the navigational hazard, consider the case of a TOPS-class spacecraft (weight 1500 lbs (675 kg); cross-sectional area 20 m^2) intersecting the densest region of ring B, while on a 4-planet Grand-Tour (JSUN) mission. From ROBERTS (1969), the trajectory for such a mission intersects ring B at an angle 29°.2 to the ring plane, at a distance 1.81 Saturn equatorial radii from the center of the planet (cf: Figures 1 and 2). The velocity of the spacecraft with respect to the planet is 30 kms sec^{-1} in the posigrade direction. Since the ring particles are circling the planet at a velocity $\sim$ 20 kms sec^{-1}, the relative spacecraft-particle velocity is $\sim$ 10 kms sec^{-1} only.

Individual spacecraft-ring particle collisions will cause negligible damage to the spacecraft; the mass of a particle is $\sim 10^{-15}$ gm, while the TOPS is designed to withstand impacts of particles of mass less than 10^{-2} gm striking at velocities $\sim$ 20 kms sec^{-1}. Collectively, spacecraft-ring particle collisions will affect the subsequent trajectory of the spacecraft; interchange of momentum will cause a small change in the spacecraft velocity. If all spacecraft-ring particle collisions are inelastic, the velocity will be reduced by 1 part in 2.5×10^5, or only 0.4 m sec^{-1}.

If the spacecraft should intersect the rings at an angle α to the ring plane, the estimates of the mass of material encountered, and of the velocity-change experienced, should both be scaled by the factor csc α. Clearly, the velocity of the spacecraft will be catastrophically reduced if the "fly-through" trajectory intersect the ring at a very shallow angle.

5. CONCLUSIONS

Past dynamical and photometric studies of the physical structure of the ring system have been subjected to detailed criticism. On the basis of that critique, the best of the available photometric data have been re-interpreted in terms of the Mie theory of scattering. A new ring model has been developed; the surface density appears to be everywhere less than unity, and the particles appear to be ice crystals of characteristic radius $\sim 0.1\ \mu$. No reliable information has been obtained concerning the probable size-distribution of the particles. The new ring model has been used to estimate the seriousness of the navigational hazard presented by the ring system. Calculations suggest that the rings are probably not a significant hazard to spacecraft. However, that conclusion cannot be considered final. Further study of the ring model is required.

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